

Last time: triple integrals

Let E be the solid bounded by the cylinder $x^2 + y^2 = 1$, the paraboloid $z = 1 - x^2 - y^2$, and the plane $z = 2$. Sketch E , and find a region D and functions $u_1(x, y)$, $u_2(x, y)$ such that

$$E = \{(x, y, z) \mid (x, y) \in D, u_1(x, y) \leq z \leq u_2(x, y)\}.$$

Discuss with your neighbour(s).

- (a) We're working on it.
- (b) We're stuck.
- (c) We have different answers and we don't know who is right.
- (d) We have the same answer.

If you're done, remember that we can find the volume of E by integrating the function $f = 1$ over E :

$$V(E) = \iiint_E dV = \iint_D \int_{u_1(x,y)}^{u_2(x,y)} dz \, dA.$$

Can you calculate $V(E)$?

Announcements

- Deadline for Midterm 2 regrade requests is Thursday.
- I-clicker participation is now optional (your i-clicker scores will only count towards your final grade **if it helps you**; otherwise they will not count).
- Quiz 4 is on Thursday.

Practice with cylindrical coordinates

Recall

$$E = \{(r, \theta, z) \mid 0 \leq \theta \leq 2\pi, 0 \leq r \leq 1, 1 - r^2 \leq z \leq 2\}.$$

Set up the integral to find the moment of inertia about the z -axis of a solid with shape E and constant density of ρ_0 .

$$\begin{aligned} I_z &= \iiint_E (x^2 + y^2) \rho_0 dV \\ &= \int_0^{2\pi} \int_0^1 \int_{1-r^2}^2 ((r \cos \theta)^2 + r \sin \theta)^2 r \rho_0 dz dr d\theta \\ &= \int_0^{2\pi} \int_0^1 \int_{1-r^2}^2 r^3 \rho_0 dz dr d\theta. \end{aligned}$$

Calculating the integral

We calculate

$$\begin{aligned}\int_0^{2\pi} \int_0^1 \int_{1-r^2}^2 r^3 \rho_0 dz dr d\theta &= \int_0^{2\pi} d\theta \int_0^1 r^3 \rho_0 [z]_{1-r^2}^2 dr \\ &= 2\pi \int_0^1 \rho_0 r^3 (r^2 + 1) dr \\ &= 2\pi \rho_0 \int_0^1 r^5 + r^3 dr \\ &= 2\pi \rho_0 \left[\frac{1}{6} r^6 + \frac{1}{4} r^4 \right]_0^1 \\ &= \frac{5}{6} \pi \rho_0.\end{aligned}$$

Practice with spherical coordinates

Consider the set

$$B = \{(\rho, \theta, \phi) \mid \rho = 1\}.$$

Can you sketch this shape? Can you write the equation in rectangular coordinates (i.e. (x, y, z))? Discuss with your neighbour(s).

- (a) We're working on it.
- (b) We're stuck.
- (c) We have different answers and we don't know who is right.
- (d) We have the same answer.