

Last time: absolute maxima and minima

Is there a point on the graph $z = \sqrt{x^2 + y^2}$ that's closest to the point $P = (4, 2, 0)$? Furthest?

- (a) There is a closest point and a furthest point.
- (b) There is a closest point but no furthest point.
- (c) There is a furthest point but no closest point.
- (d) There is neither a furthest point nor a closest point.
- (e) I don't know.

Recall: the Extreme Value Theorem

Let $f : D \rightarrow \mathbb{R}$ be a continuous function on D , which is closed and bounded. Then f attains a maximum value on D at some point $P \in D$, and either

- P is on the boundary of D , OR
- P is a critical point of f .

Today: How can we find the maximum value of f along the boundary of D , without checking every single point?

Practice with Lagrange multipliers

We have the following three equations:

$$2x = \lambda 2x$$

$$2y = -\lambda 2y$$

$$x^2 + y^2 = 4.$$

How many solutions (x, y, λ) are there?

- (a) No solutions.
- (b) 2.
- (c) 4.
- (d) Infinitely many.
- (e) I don't know.

Lagrange multipliers (in three variables)

Assume f, g are functions of three variables with continuous first order partial derivatives.

If $f(x_0, y_0, z_0)$ is the maximum value of f over the level surface $g(x, y, z) = k$, then either

- $\nabla g(x_0, y_0, z_0) = \langle 0, 0, 0 \rangle$ OR
- $\nabla f(x_0, y_0, z_0) = \lambda \nabla g(x_0, y_0, z_0)$, for some $\lambda \in \mathbb{R}$.

The same theorems hold for minimum values too.

Practice with Lagrange multipliers

Suppose f, g are functions on \mathbb{R}^3 with continuous first order partial derivatives.

Suppose that f achieves its maximum over the set $\{g(x, y, z) = k\}$ at the point P . Which of the following is **not** possible?

- (a) $\nabla f(P) = \langle 0, 0, 0 \rangle, \nabla g(P) = \langle 0, 0, 0 \rangle$.
- (b) $\nabla f(P) = \langle 0, 0, 0 \rangle, \nabla g(P) = \langle 1, 3, -2 \rangle$.
- (c) $\nabla f(P) = \langle 4, 0, 1 \rangle, \nabla g(P) = \langle 0, 0, 0 \rangle$.
- (d) $\nabla f(P) = \langle -2, -6, 4 \rangle, \nabla g(P) = \langle 1, 3, -2 \rangle$.
- (e) $\nabla f(P) = \langle 4, 0, 1 \rangle, \nabla g(P) = \langle 1, 3, -2 \rangle$.

Maximizing the volume of a box

Which of the following is true?

- (a) D is closed and bounded, so by the extreme value theorem, f has a maximum and a minimum on D .
- (b) D is not closed or bounded, so we can't say anything.
- (c) D is not closed or bounded, but we can argue for geometric reasons that f has a maximum and a minimum on D .
- (d) D is not closed or bounded, but we can argue for geometric reasons that f has a maximum but not a minimum.