MATH 595 Thursday 29 March

Review on abstract curves; divisors on curves

(1) **Exercise I.6.1**

Recall that we say that a curve is *rational* if it is birationally equivalent to $\mathbb{P}^1$. Let Y be a non-singular rational curve which is not isomorphic to $\mathbb{P}^1$; let K be the function field of Y.

(a) Prove that Y is isomorphic to an open subset of $\mathbb{A}^1$.

(b) Conclude that Y is affine, and that its ring of functions is a UFD.

(2) **Exercise I.6.4**

Let Y be a non-singular projective curve over k with function field K. Show that every non-constant rational function f on Y (i.e. $f \in K \setminus k$) defines a surjective morphism $\phi : Y \to \mathbb{P}^1$ such that for every $P \in Y$, $\phi^{-1}(P)$ is finite.

(3) Let X be a curve with function field K. Show that for $f, g \in K^*$, the corresponding principal divisors satisfy

$$(f/g) = (f) - (g).$$

(4) Let $f : X \to Y$ be a finite morphism of non-singular curves. We defined a homomorphism $f^* : \mathrm{Div}(Y) \to \mathrm{Div}(X)$. Show that f^* preserves linear equivalence, and hence induces a homomorphism of the divisor class groups.