MATH 402 Review for September 10–14

Topics: Triangle similarity results (2.5); the Parallel Postulate is equivalent to Playfair's Postulate; some facts about "area" (2.4.1); angle sum results for triangles. A few things about hyperbolic geometry (1.7). These were covered in lecture, on Worksheet 2, and in the Project (1.7). This material will also appear in Homework 3.

1. **Recall from last week:**

   (a) The following statement holds in neutral geometry (i.e. we don't need the parallel postulate to prove it): *Suppose that a line n crosses two lines ℓ and m such that any of the following hold:*

   - a pair of alternate angles is congruent;
   - a pair of corresponding angles is congruent;
   - the sum of two interior angles on the same side of n is equal to $180°$.

   Then ℓ and m are parallel.

   The converse statement is true in Euclidean geometry.

   (b) SAS congruence is an axiom in Hilbert's axiomatic system; the other triangle congruence results (SSS, ASA, AAS) are theorems. They all hold in neutral geometry.

   (c) Make sure you remember Pasch's Axiom.

2. **Key facts about area:**

   (a) Make sure you know how to compute the area of a rectangle, a triangle, or a parallelogram.

   (b) If you cut a shape up into disjoint pieces, the area of the whole shape is the sum of the area of the pieces.

   (c) If two figures are congruent, they have the same area.

3. **Things to know about triangle similarity (all results are only in *Euclidean* geometry!):**

   (a) Know the definition of *similarity* for triangles.

   (b) We had three theorems, labelled (1), (2), and (3): (3) told us that if a line ℓ line passes through two sides of a triangle cutting off segments which are proportional to the original side lengths (or to the complementary segments), that line must be parallel to the third side of the triangle; and (1) and (2) told us that the converse is true too.

   (c) We used these three theorems to prove AAA similarity. Then we could also prove SAS similarity (and on Homework 3 you'll prove SSS similarity).

4. **Playfair's Postulate is equivalent to the Parallel Postulate:**

   (a) Know that this is true!

   (b) Review the proofs of each direction of the equivalence.

5. **Things about triangles:**

   (a) In Euclidean geometry, the three angles of a triangle always sum to $180°$.

   (b) In neutral geometry, we can't prove this, but we can prove that the angle sum is $\leq 180°$.

   (c) In Euclidean geometry, we can use similar triangles to prove Pythagoras's theorem.

Practice Questions

1. **Make sure you finish the worksheet**
    - I know this week's was longer than last week's, but try to finish all the questions on your own. Feel free to work in groups, and ask me for help if you get stuck.
2. **Practice with similar triangles**
    - There are lots of resources online to help high school students practice with these concepts. If you want to make sure you've got the basics down, you can try a few of these. For example, go to `https://www.ixl.com/math/geometry` and try some of the exercises in Section P, Similarity. (Or just google "similar triangles worksheet" to get many many examples.)
3. **Playfair's Postulate is equivalent to the Parallel Postulate:**
    - Review the proofs that Playfair's Postulate implies the Parallel Postulate, and conversely that the Parallel Postulate implies Playfair's Postulate. Is one of them easier for you to understand and remember than the other? Can you think what makes it easier?