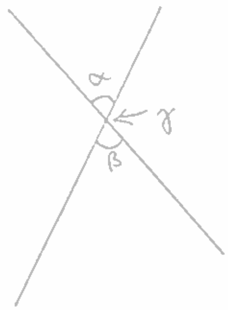
Homework # 2 . Math 402 Fall 2018.

Exercise 1.

(a) Draw a picture illustrating the Supplementary Angle Theorem.

$m\angle\alpha + m\angle\beta = 180°$

(b) Prove the vertical angle theorem.



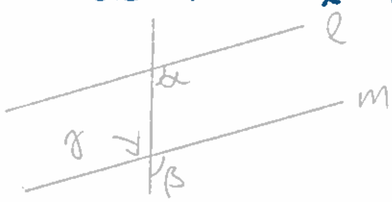
· WTS $\angle\alpha \cong \angle\beta$, or equivalently $m\angle\alpha = m\angle\beta$

SAT $\Rightarrow$ · $m\angle\alpha + m\angle\gamma = 180°$

· $m\angle\beta + m\angle\gamma = 180°$

$\therefore$ $m\angle\alpha = m\angle\beta = 180° - m\angle\gamma$

(c) i. Assume a line n falls across a lines ℓ and m s.t. the corresponding angles are congruent. Prove that ℓ is parallel to m.

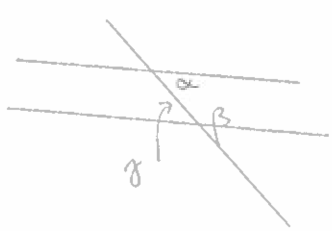


We know that $\alpha \cong \beta$

Now by the V.A.T. $\beta \cong \gamma$

$\therefore$ $\alpha \cong \gamma$, and by Prop 27 $\ell \parallel m$.

ii. Assume n falls across ℓ and m s.t. the sum of the measures of the interior angles on the same side is two right angles. Prove that ℓ is parallel to m.



· We know that $m\angle\alpha + m\angle\beta = 180°$

· By S.A.T we also know that

$m\angle\beta + m\angle\gamma = 180°$

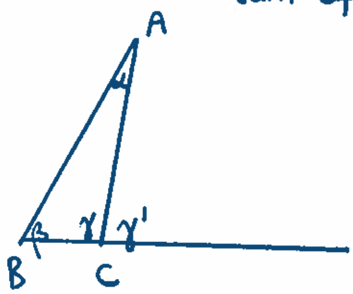
$\therefore$ $m\angle\alpha = m\angle\gamma$, so $\alpha \cong \gamma$

Then by Prop 27. $\ell \parallel m$.

(d) (Exercise 1 continued)

Let ΔABC be a triangle.

Use the E.A.T. and the S.A.T. to show that the sum of any two interior angles is less than 180°.



By the Exterior angle theorem,

$m\angle\gamma' > m\angle\alpha,\ m\angle\beta.$

By the supplementary angle theorem,

$180° = m\angle\gamma' + m\angle\gamma$

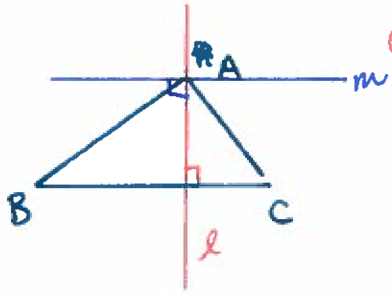
$\therefore\ 180° > m\angle\alpha + m\angle\gamma$

and $180° > m\angle\beta + m\angle\gamma$

By extending a different side we can show that $m\angle\alpha + m\angle\beta < 180°$ too.

Exercise 2 Consider a triangle ΔABC.

(a) Show that you can construct a parallel to $\overleftrightarrow{BC}$ through the point A.

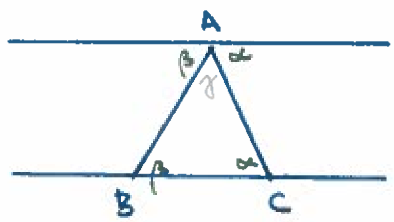


① $\exists$ a line l through A perpendicular to $\overleftrightarrow{BC}$.

② $\exists$ a line m through A perpendicular to l.

Now by Prop 27 (or part (c) of Exercise 1) m is parallel to $\overleftrightarrow{BC}$

(b) Use this to show that Playfair's postulate implies that the sum of angles in a triangle is equal to 180°.

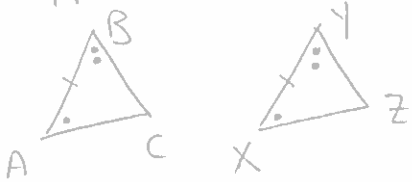


By Prop. 29 (which uses Playfair's postulate) we have congruent alternate angles α at A and C and β at A and B

By the supplementary angle theorem $m\angle\alpha + m\angle\beta + m\angle\gamma = 180°$ as claimed.

Exercise 3: (a) Prove that SAS congruence implies ASA congruence.

Suppose we have two triangles:



we need to show that

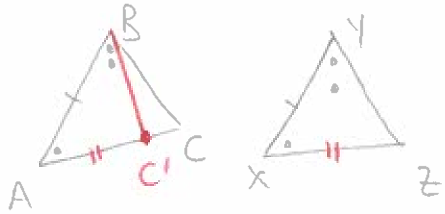
$\triangle ABC \cong \triangle XYZ$.

Since we know SAS congruence, it is enough to show that $\overline{AC} \cong \overline{XZ}$

Suppose towards a contradiction that $\overline{AC} \not\cong \overline{XZ}$.

WLOG assume $AC > XZ$.

Then we can choose a point C' on $\overline{AC}$ s.t. $\overline{AC'} \cong \overline{XZ}$.

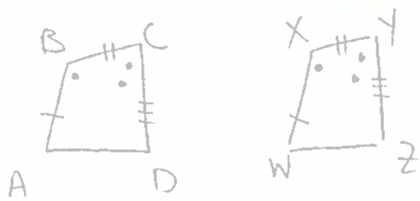


Now by SAS, $\triangle ABC' \cong \triangle XYZ$.

In particular, $\angle ABC' \cong \angle XYZ$, which contradicts the fact that

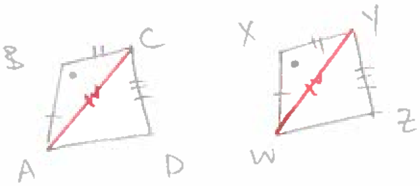
$\angle ABC \cong \angle XYZ$.

(b) State and prove a SASAS congruence relation for quadrilaterals:



" Given two quadrilaterals and a correspondence between them such that three pairs of sides are congruent, and the two pairs of included angles are congruent, then the quadrilaterals are congruent "

proof Draw line segments $\overline{AC}$ and $\overline{WY}$



- By SAS, $\triangle ABC \cong \triangle WXY$

So $\overline{AC} \cong \overline{WY}$

- Also, because $\angle BCD \cong \angle XYZ$ by assumption and $\angle BCA \cong \angle XYW$ by SAS,

we have $\angle ACD \cong \angle WYZ$.

$\therefore$ By SAS again, $\triangle ACD \cong \angle WYZ$

We can see that $\overline{AD} \cong \overline{WZ}$, and $\angle ADC \cong \angle WZY$.

Also $\angle BAD \cong \angle XWZ$ (by combining the two SAS results and adding the angles.)