Workshop on Mathematical Billiards: 2019

Sydney

24 – 27 June 2019

Program and Abstracts

Organisers:
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Workshop on Mathematical Billiards

The University of Sydney

24–27 June 2019

Lecture Theatre 024, New Law Building Annex

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Thursday, 27 June

Session 1		Chair: Leonid Bunimovich
9:30–10:20	Vered Rom-Kedar	**On some nearly separable impact systems**
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14:00–14:50	Vera Roshchina	**Invisibility in billiards**
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Chaotic billiards with leaks.

Eduardo Altmann. The University of Sydney, Australia

When using mathematical billiards to describe physical settings one often needs to take into account that the confinement is not perfect and that trajectories (or energy) can escape. In this talk I will discuss how to apply and extend the theory of open dynamical systems to billiards with holes or leaks. In particular, I will focus on the case of optical billiards, in which light rays can be partially reflected and partially transmitted. I will then show how a mathematical description of chaotic dynamics in these systems can be obtained using conditionally-invariant measures and a generalized Perron-Frobenius operator.

PHYSICAL VERSUS MATHEMATICAL BILLIARDS: from chaos to order and back.

Leonid Bunimovich. Georgia Tech, USA

In standard mathematical billiards a point particle moves in a billiard table. If instead one considers a moving hard ball (physical particle) then there are two possibilities. Either this ball is rough or smooth. In case when a ball is rough then it acquires rotational degree of freedom upon reflection which leads to very difficult problems even in the simplest case of so called no-slip billiards. Motion of a single smooth ball was never considered though apparently because for all most basic cases (circle, triangle, Sinai billiards, stadium) dynamics is completely similar to the one of mathematical billiards. I will show on simple examples that in fact anything is possible in such transitions, i.e. a chaotic billiard may become non-chaotic and vice versa. Consideration of such physical billiards already led to some quite surprising results in quantum chaos and statistical mechanics.

Renormalization in Low-dimensional Dynamics.

V.V.M.Sarma Chandramouli. Department of Mathematics Indian Institute of Technology Jodhpur India

Renormalization is a method to study microscopic geometrical properties of attractors. This microscope is an operator on some space of one-dimensional systems. Given a one-dimensional system its renormalization is a similar system, which describes the dynamics at a smaller scale. Infinitely renormalizable systems are the ones, for which one can repeatedly apply the renormalization operator and study the dynamics at arbitrarily small scales. In this talk we discuss the main result that hyperbolicity of renormalization in C^2 space breaks down althrough there is still slow convergence to the renroalization fixed point. There is no universality and regidity, when the smoothness gets below C^2. Next, in Henon renromalization, we discribe how one-dimensional Cantor set deforms into the Cantor set of conservative map. To understand this deformation we study the invarient line field which is carried on the Cantor set. This shows the complexity of the geometry of these Cantor sets. Finally, it raises to a question: Is there any Billiard flow along these invarient line field?

Integrable Billiards in Euclidean and Pseudo-Euclidean Spaces and Classical Extremal Polynomials.

Vladimir Dragović. UT Dallas

We describe periodic and elliptic periodic trajectories of billiards within quadrics in Euclidean and Pseudo-Euclidean d-dimensional Spaces

in algebro-geometric terms and also in terms of polynomial-functional equations. Connections with classical extremal polynomails on d intervals lead to combinatorial classifications of such trajectories. In the planar cases we observe in addition relationship with discriminantly

separable polynomials. The results are joint with Milena Radnovic, while those related to the Minkowski plane are also joint with Anani Adabrah.

Billiard flow in nibbled ellipses.

Krzysztof Fraczek. Nicolaus Kopernikus University, Poland

I plan to present some basic properties of the billiard flow on nibbled ellipses. The boundary of a nibbled ellipse consist of a chain of elliptic and hyperbolic arcs, all coming from confocal conics. The main aim of the talk is to present some steps and tools in the proof of equidistribution for almost all invariant sets determined by caustics of the billiard.

On curves with Poritsky property and Liouville nets.

Alexey Glutsyuk. CNRS (ENS de Lyon) and HSE University (Moscow).

To each planar convex closed curve C the string construction associates a family of bigger closed curves G(t) whose billiards have C as a caustic. Each curve G(t) induces a dynamical system on C: given two tangent lines to C through the same point in G(t), the tangency point of the left tangent line is sent to the tangency point of the right tangent line. A curve C is said to have Poritsky property, if this dynamical system has an invariant length element on C that is the same for all t. Curves with Poritsky property are closely related to integrable billiards. H.Poritsky have shown that the only planar curves with Poritsky property are conics. We extend his result to surfaces of constant curvature. We consider curves with Poritsky property on arbitrary Riemannian surface and obtain the following results: 1) a formula for the invariant length element in terms of the geodesic curvature and the natural length parameter; 2) each germ of curve with Poritsky property is completely determined by its 4-jet.

We also present the following very recent joint result with Sergei Tabachnikov and Ivan Izmestiev. Given a curve C bounding a topological disc on a Riemannian surface. Then the following statements are equivalent:

1) The curve C has Poritsky property.

2) The corresponding family of string constructions has Graves property: smaller curves are caustics for bigger curves.

3) The metric on the concave side of C is Liouville.

Periodic billiard trajectory and Morse theory on loop spaces.

Kei Irie. The University of Tokyo, Japan

As is classically known, applying Morse theory to the length functional on the free loop space of a closed Riemannian manifold, one can show the existence of closed geodesics from non-vanishing of (relative) loop space homology. I will first explain how this well-known result generalizes to the case of periodic billiard trajectories. Then, based on this result and inspired by known constructions in symplectic geometry, I will introduce an invariant called 'capacity' of billiard tables, and explain some applications of this invariant. If time permits, I will also discuss a series of infinitely many capacities which are defined from S^1-equivariant homology of the free loop space.

Painlevé Dynamics.

Nalini Joshi. The University of Sydney, Australia

The solutions of Painlevé and discrete Painlevé equations are closely related to elliptic functions. Their initial value spaces can be regularised after blowing up 8 points in $\mathbb{P}^1 \times \mathbb{P}^1$, in the same way as the corresponding spaces of elliptic functions. I will give a survey of the geometric theory developed by Sakai (2001) before considering a new example that turns out to have a different structure to those described by Sakai. (The talk covers joint work with many collaborators, including James Atkinson, Philip Howes, Nobutaka Nakazono and Milena Radnovic.)

Integrable billiard books model the base of Liouville foliations.

Irina Kharcheva. Lomonosov Moscow State University, Moscow, Russia.

Let us consider a free motion of a particle in some fixed domain $\Omega \subset \mathbb{R}^2$ with elastic reflection at the boundary $P = \partial\Omega$. We obtain a Hamiltonian dynamical system with a Hamiltonian that equals to the scalar square of the velocity vector. The dynamical system is called a billiard.

If the domain's boundary P is a piecewise curve and consist of several arcs of confocal ellipses and hyperbolas then the billiard has a following special property: the straight lines containing the segments of the polygonal billiard trajectory are tangents to a certain quadric (ellipse or hyperbola). The parameter of this quadric is the value of the additional integral Λ. Thus this billiard is integrable and called an elementary billiard.

A billiard book is a generalization obtained by gluing of elementary billiards along their boundaries. A billiard book is still an integrable Hamiltonian system.

We can describe any integrable Hamiltonian system with 2 degrees of freedom in terms of Fomenko-Zieschang invariants: atoms, coarse molecules and mark molecules (see [1]). Such invariants allow us to speak about the equivalence between closures of trajectories of different dynamical systems.

Researching billiard books we try to model famous integrable systems in terms of Fomenko-Zieschang invariants (see, for example, [2]). It turns out that a coarse molecule (a base of the Liouville foliation) of any integrable Hamiltonian systems with 2 degrees of freedom is equivalent to a coarse molecule of some billiard book. Details of this result will be presented.

This work was supported by the program "Leading Scientific Schools" (grant no. NSh-6399.2018.1, Agreement No. 075-02-2018-867).

1. *A. T. Fomenko , H. Zieschang* A topological invariant and a criterion for the equivalence of integrable hamiltonian systems with two degrees of freedom // *Mathematics of the USSR - Izvestiya.* – 1991. – Vol. 36, no. 3. – P. 567-596.

2. *V. V. Vedyushkina* and *I. S. Kharcheva.* Billiard books model all three-dimensional bifurcations of integrable Hamiltonian systems. Sbornik: Mathematics, 209(12), 1690—1727.

Magnetic billiards on the sphere and hyperbolic plane.

Andrey Mironov. Novosibirsk State University, Russia

We consider billiard ball motion in a convex domain on a constant curvature surface influenced by the constant magnetic field and discuss the existence of integral of motion which is polynomial in velocities. We prove that if such an integral exists then the boundary curve of the domain determines an algebraic curve which must be nonsingular. From here it follows that for any domain different from round disc for all but finitely many values of the magnitude of the magnetic field billiard motion does not have Polynomial in velocities integral of motion. Results were obtained with Misha Bialy, Tel Aviv University.

Two Questions on Convex Billiards.

Sônia Pinto de Carvalho, Departamento de Matemática, ICEx, Universidade Federal de Minas Gerais, Brasil

In this talk I would like to pose two questions on convex billiards.

First, I will look at the circular billiard and address the question: is there any surface where the geodesic circular billiard is not integrable?

Then, I will present properties of the instability zone of convex billiards on the plane, containing the 2-periodic orbits, and ask if there is a curve where this instability zone is ergodic.

Integrable Flat Billiards with Potential.

Sergey Pustovoytov, Moscow State University, Russia

Let us consider a flat billiard system in the domain bounded by the arcs of confocal quadrics (ellipses or hyperbolas). Every angle is equal to $\frac{\pi}{2}$. The Hooke potential is placed at the common center of these quadrics. The potential attracts or repulses the material point. Such a dynamic system on the symplectic phase manifold $M^4 = \{x, y, \dot{x}, \dot{y}\}$ is appear to be a Hamiltonian integrable system. Hamiltonian is H, second first integral is G.

$$H = \frac{k(x^2 + y^2)}{2} + \frac{\dot{x}^2 + \dot{y}^2}{2} \tag{1}$$

$$G = \frac{\dot{x}^2}{a} + \frac{\dot{y}^2}{b} + \frac{(x\dot{y} - \dot{x}y)^2}{ab} - k(1 - \frac{x^2}{a} - \frac{y^2}{b}) \tag{2}$$

These functions are functionally independent. Also the condition $\{H, F\} = 0$ is preserved.

Let us consider the isoenergy manifold $Q^3 = \{x \in M^4 : H(x) = h\}$ for every regular value of Hamiltonian h. According Liouville theorem, Q^3 fibered by surfaces of levels of integrals. If level is regular, corresponding surface is homeomorphic to join of Liouville tori. In order to describe bifurcations of these tori through singular level we use concept of *atom* introduced by Anatoly Fomenko in [1]. We describe the topology of Liouville foliation of Q^3 using invariants of Fomenko-Zieschang (*marked molecules*). Two systems are Liouville equivalent if and only if their marked molecules are coincident.

In these terms we calculated marked molecules for every described billiard for every Q^3 using the bifurcation complexes. Then we matched billiard systems with other Liouville equivalent systems.

1. A.V.Bolsinov, A.T.Fomenko. Integrable Hamiltonian systems. Geometry, topology, classification. Chapman & Hall/CRC, Boca Raton (2004).

2. Kozlov V.V. Some integrable generalizations of the Jacobi problem on geodesics on an ellipsoid // J. Appl. Math. Mech., 59:1 (1995)

3. Kharlamov M.P. Topological analysis and Boolean function: I. Methods and applications to the classical systems //Non-lineal dynamics, 6:4, p. 769-805

4. V. V. Fokicheva, "A topological classification of billiards in locally planar domains bounded by arcs of confocal quadrics", Sb. Math., 206:10 (2015), 1463–1507

On some nearly separable impact systems.

Vered Rom-Kedar. Weizmann Institute, Israel

Near-integrability is usually associated with smooth small perturbations of smooth integrable systems. We show that studying integrable mechanical Hamiltonian flows with impacts that respect the symmetries of the integrable structure provide an additional rich class of non-smooth systems that can be studied by energy-momentum maps and perturbation methods [1]. Moreover, the analysis can be extended to systems with soft steep potentials that limit to the impact systems. For example, for some of these systems, we show that KAM theory may be applied, proving that for a large portion of phase space the perturbed motion is conjugate to rotations on a torus [2]. On the other hand, other simple impact systems have inherently non-rotational motion – we show cases in which the motion on level sets is conjugate to directed motion on L shaped polygonal billiards [3].

Based on the following joints works:

1. M. Pnueli and V. Rom-Kedar, On the structure of Hamiltonian impact systems, preprint https://arxiv.org/abs/1903.0885

2. M. Pnueli & V. Rom-Kedar On near integrability of some impact systems, SIAM-DS, 17(4), 2707—2732, 2018, https://doi.org/10.1137/18M1177937

3. L. Becker, S. Elliott, B. Firester, S. Gonen Cohen, M. Pnueli & V. Rom-Kedar, in preparation.

Invisibility in Billiards.

Vera Roshchina, UNSW Sydney

An object can be disappeared from a specific viewpoint or direction by the use of mirror surfaces. The problem of designing such mirror cloaks with specific invisibility properties can be modelled by a billiard in an unbounded domain. It is possible to construct bodies invisible from finitely many viewing points and directions, however, the natural assumptions on the object's structure (such as piecewise smoothness) restrict the possibilities for such constructions. In this talk I will survey some old and new results in this area.

The talk is based on joint work with Alexander Plakhov (University of Aveiro, Portugal).

Gutkin billiard tables.

Lior Shalom. Tel Aviv University, Israel

I will first explain Gutkin billiards, his result and conjecture in "Capillary floating and the billiard ball problem". Next I will elaborate on a joint work with Misha Bialy and Andrey Mironov. I will discuss the so called outer billiards and present our theorem that total integrability cannot happen between two invariant curves. I will also present a simulation showing a chaotic area. If the time will allow it I will elaborate on my current work with Misha Bialy - finding magnetic billiards satisfying Gutkin property.

Towards Fourier's law of heat conduction in a fast-slow deterministic heat conduction model with billiards.

Imre Peter Toth. Budapest University of Technology and Economics, Hungary

I discuss a deterministic model of heat conduction, first introduced in 1992 by Bunimovich, Liverani, Pellegrinotti and Suhov, and later studied by in 2008 by Gaspard and Gilbert. It consists of billiard disks confined to positions near lattice sites, such that each particle can exchange energy with its neighbours. In the rare interaction limit, when energy exchange collision happen seldom, the evolution of the energies is given by a Markov jump process. This Markov process is itself a (stochastic) model of heat conduction, which seems to obey Fourier's law.

I discuss our heuristic understanding of heat conduction in these models, supported by numerical studies, as well as the few rigorous results that we have concerning the convergence to a Markov process in the rare interaction limit.

Join work with P. Balint, T. Gilbert, P. Nandori and D. Szasz.

The topological effects and constructions modeled by integrable billiards.

Viktoria Vedyushkina, Lomonosov Moscow State University, Russia

Consider an integrable billiard in a planar domain bounded by arcs of confocal ellipses and hyperbolas. Let us consider a domain bounded by arcs of the focal line, an ellipse and two hyperbolas. Glue n copies of such billiard along two convex boundary arcs and the segment of the focal line. Let us define the motion as follows: when hitting a boundary arc, the billard ball will change the domain by some permutation. Using this construction (which is called billiard book) we can model interesting effects in the topology of the foliations on the isoenergy 3-dimensional manifolds of integrable Hamiltonian systems. The first there exist permutations that, being attributed to the convex arcs and the segment of the focal straight line, make it possible to obtain a foliation in the neighborhood of an unstable trajectory any non-degenerate saddle bifurcation of Liouville tori. The second, if one of this permutation is cyclic permutation and the permutation on the adjacent arc is the some degree k of the given cyclic permutation then the isoenergy 3-manifold is homeomorphic to the lens space L(n,k). This lens space is stratified into levels of the additional first integral which is the parameter of the confocal quadric (called caustic) tangent to all straight lines containing the links of the billiard path. As in the planar case, the motion along convex boundary arcs is stable, and the motion along the focal line segment is unstable. Moreover we show the constructions of the integrable billiards which model the isoenergy manifolds of the typical integrable case of the rigid body dynamics.

This work was supported by the program “Leading Scientific Schools” (grant no. NSh-6399.2018.1, Agreement No. 075-02-2018-867)