SYDNEY CONFERENCE TALKS

Wednesday, June 12

Sanju Velani (University of York, UK)

Diophantine Approximation on manifolds: lower bounds for Hausdorff dimension.

There has recently been much activity in the theory of Diophantine approximation in which the points of interest are restricted to a sub-manifold M. I plan to provide a review of this activity. The goal is to describe a new approach based on the Mass Transference Principle which yields sharp lower bounds for the dimension of well approximable sets restricted to M.

Seonhee Lee (Seoul National University, Korea)

Doubly metric inhomogeneous Diophantine approximation

We will briefly review the joint work with D. Kim, Y. Bugeaud and M. Rams on the Hausdorff dimension of the epsilon badly approximable vectors and the property of a vector being singular on average. Then we will present a doubly-metric version of the above Hausdorff dimension of epsilon badly approximable vectors. The latter part of the talk is a joint work with Wooyeon Kim.

Alina Ostafe (UNSW, Australia)

Multiplicative relations in orbits of polynomial dynamical systems

The underlying motif of this talk is showing finiteness of multiplicative relations between elements in orbits of algebraic dynamical systems over number fields. After surveying several finiteness recent results about multiplicative relations over the cyclotomic closure of a number field, we will concentrate on the more general case of multiplicative dependence modulo a finitely generated multiplicative subgroup of the field. We present a series of results, many of which may be viewed as a blend of Northcott's theorem on boundedness of preperiodic points and Siegel's theorem on finiteness of solutions to S-unit equations. We also outline some open questions along the talk.

This is joint work with Attila Berczes, Min Sha, Igor Shparlinski, Joseph Silverman and Umberto Zannier (in different subsets).

Bing Li (SCUT, China)

Denseness of intermediate beta-shifts of finite type

We determine the structure of the set of intermediate β-shifts of finite type. Specifically, we show that this set is dense in the parameter space

$$\Delta := \{(\beta, \alpha) \in \mathbb{R}^2 \colon \beta \in (1, 2) \text{ and } 0 \le \alpha \le 2 - \beta\}.$$

This generalises the classical result of Parry from 1960 for greedy β-shifts.

This is a joint work with Tuomas Sahlsten, Tony Samuel and Wolfgang Steiner.

Thursday, June 13

Alexander Gorodnik (University of Zürich, Switzerland)

Littlewood conjecture and friends

We discuss several problems in Diophantine approximation that involve minimizing products of linear forms computed at integral points and explain how these questions can be studied through analyzing behavior of orbits for suitable group actions.

Michael Björklund (Chalmers, Sweden)

Approximate lattices - yesterday, today and perhaps tomorrow?

While both lattices and approximate groups (in various disguises) have been subject to intense studies over the years, their natural merger (approximate lattices) is much less known. First conceived by Yves Meyer in late 60'ies under the (not too appealing) name ”relatively dense harmonious sets”, approximate lattices in Euclidean spaces - as well as in general locally compact abelian groups - later coined ”quasicrystals” - were intensely studied in the 90'ies, and early 00'ies, from various angles. Only very recently (around 2017) were approximate lattices in arbitrary locally compact groups introduced and studied. Since then, some foundations have been laid, and it is the aim of this talk to survey these - and give some suggestions what the next steps could be...

Based on joint works with Tobias Hartnick (Karlsruhe), Thierry Stuhlemeijer (Giessen), Felix Pogorzelski (Leipzig).

John Griesmer (Colorado School of Mines, USA)

Recurrence and rigidity in measurable dynamics

A set of integers S is a set of recurrence if for every measure preserving transformation (X, m, T) and every subset A of X having positive measure, there is an element n of S such that A and $T^n(A)$ have nonempty intersection. Famous examples of sets of recurrence include the set natural numbers and the set of perfect squares. Extending a technique of Alan Forrest, we will construct a set of integers S such that every translate $S + k$ of S is a set of recurrence and every translate $S + k$, when enumerated as $(s_n + k)$, is a rigidity sequence for some weak mixing system. We will pose a problem as to whether every set of recurrence can be obtained by a similar construction.

Mumtaz Hussain (University of Latrobe, Australia)

Metrical theory for improvements to Dirichlet's theorem

In this talk, I will discuss the metrical theory associated with the set of Dirichlet non-improvable numbers. Let $\Psi : [1, \infty) \to \mathbb{R}_+$ be a non-decreasing function, $a_n(x)$ the n'th partial quotient of x and $q_n(x)$ the denominator of the n'th convergent. The set of Ψ-Dirichlet non-improvable numbers

$$G(\Psi) := \Big\{ x \in [0,1) : a_n(x)a_{n+1}(x) \ > \ \Psi\big(q_n(x)\big) \text{ for infinitely many } n \in \mathbb{N} \Big\},$$

is related with the classical set of $1/q^2\Psi(q)$-approximable numbers

$$\mathcal{K}(\Psi) := \left\{ x \in [0,1) : \left| x - \frac{p}{q} \right| < \frac{1}{q^2\Psi(q)} \text{ for infinitely many } (p,q) \in \mathbb{Z} \times \mathbb{N} \right\},$$

in the sense that $\mathcal{K}(3\Psi) \subset G(\Psi)$. In this talk, I will explain that the Hausdorff measure of the set $G(\Psi)$ obeys a zero-infinity law for a large class of dimension functions. Together with the Lebesgue measure-theoretic results established by Kleinbock & Wadleigh (2016), our results contribute to building a complete metric theory for the set of Dirichlet non-improvable numbers.

Another recent result that I will discuss will be the Hausdorff dimension of the set $G(\Psi) \setminus \mathcal{K}(3\Psi)$.

These are joint works with Kleinbock-Wadleigh-Wang (2018) and Bakhtawar–Bos (2019).

Friday, June 14

Dmitry Kleinbock (Brandeis, USA)

Uniform approximation in ergodic theory with shrinking targets

Let (X, μ, T) be a finite measure-preserving system, and let $\{B_n\}$ be a sequence of nested subsets of X. Following a recent paper by Dubi Kelmer, let us say that $x \in X$ eventually always hits $\{B_n\}$ if for every sufficiently large N there exists $n \in \{1, \ldots, N\}$ such that $T^n x \in B_N$. This is a uniform analogue of the hitting property usually considered in ergodic theory with shrinking targets via dynamical Borel-Cantelli lemmas. For circle rotations this corresponds to uniform inhomogeneous Diophantine approximation.

In general not much can be said about the magnitude of the sets of eventually always hitting points. In this work, joint with Florian Richter and Ioannis Konstantoulas, we focus our attention on systems where translates of targets exhibit near perfect mutual independence. For such systems we present tight conditions on the shrinking rate of the targets so that the set of eventually always hitting points is null or co-null.

Michael Coons (University of Newcastle, Australia)

Scaling of the diffraction measure of k-free integers

Asymptotics are derived for the scaling of the total diffraction intensity for the set of k-free integers near the origin, which is a measure for the degree of patch fluctuations. This is joint work with Michael Baake (Bielefeld, Germany).

Sam Chow, Oxford

Lonely runners in function fields

The lonely runner conjecture (1967) is an extremal problem related to Dirichlet's approximation theorem. With Luka Rimanic, we introduce a function field analogue and obtain some partial results.

Johannes Schleischitz (Middle East Technical University, Cyprus)

Diophantine approximation in Cantor sets

In 1984, K. Mahler asked how well irrational numbers in his Cantor middle-third set can be approximated by a) rational numbers inside the Cantor middle-third set b) rational numbers outside of it. Only in the last couple of years this particular question has been addressed in a few papers, in particular two papers by Fishman and Simmons, in context of more general fractal sets (including all ?missing digit Cantor sets?). After an exposition of the mentioned results, we generalize and improve some of them. We further answer a question raised by Fishman and Simmons, showing that any missing digit Cantor set contains an irrational numbers with almost all its convergents contained in it (joint work with D. Roy). We also address the question of the algebraic structure of rational numbers in missing digit Cantor sets, thereby we obtain an improvement of a result of Bloshchitsyn from 2015, and discuss several other related topics.